Cooperative Diversity of Wireless Networks with Multiple Amplify-and-Forward Relays and Hard-Decision Detections

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Abstract

This paper investigates the diversity performance of wireless cooperative networks with multiple parallel relays communicating with the destination over orthogonal channels, and focuses on the amplify-and-forward relaying protocol. The networks under consideration employ two signal-to-noise ratio (SNR) thresholds and multiple hard-decision detections (HDDs) at the destination. One SNR threshold is used to select transmitting relays in the second phase: a relay retransmits to the destination if its received SNR is larger than the threshold, otherwise, it remains silent. The other threshold is used at the destination for detection: the destination makes a hard decision on the received signal from a relay if its SNR is higher than the threshold, otherwise, the destination makes an erasure decision. Then the destination simply combines all the hard decision results and makes the final binary decision based on majority voting. The paper derives the end-to-end bit error and outage probabilities, and presents the diversity analysis of the proposed method. It is shown that the full diversity order can be achieved by setting appropriate thresholds even when the destination does not know the exact or average SNRs of the source-relay links. Simulation results corroborate our analysis. The results show that the error performance with HDDs is improved gradually as the number of relays increases.

Keywords—Wireless relay network, cooperative diversity, amplify-and-forward, outage probability, diversity analysis.

1. Introduction

In most existing wireless communication networks, cable-powered base stations can be easily equipped with spatially separated multiple antennas, while mounting multiple antennas in portable mobile terminals is not so practical because of their small-size and limited processing power. Hence, how to fully exploit the diversity benefit of multiple-antenna systems in distributed wireless communication networks has become an important issue. Recently, the concept of cooperation in wireless communications has drawn much research attention due to its potential in improving the efficiency of wireless networks [1]- [3]. In cooperative communications, the users can cooperate to relay each other's information signals, create a virtual array of transmit antennas, and hence achieve spatial diversity. Cooperative diversity technique can dramatically improve the reliability of signal transmission from each user. In general, relaying transmission strategies can be divided into two main categories: amplify-and-forward (AF) and decode-and-forward (DF). In AF protocol, relays just amplify the signal received from the source and retransmit to the destination or the next node. On the other hand, for the DF protocol, the relays decode the signal and remodulate it before retransmitting the detected version to the next node. In addition, DF protocol can be combined with coding techniques and thus forming the so-called coded cooperation [4]. Based on AF protocol, a cooperative strategy, referred to as distributed space-time coding, has also been proposed and developed [3], [5].

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Without performing any decoding, AF leads to low-complexity relay transceivers as well as lower power consumption. The AF protocol can be further divided in two categories, namely channel state information-assisted AF relaying and fixed-gain relaying. For these two AF strategies, outage and error performance have been extensively investigated [6]-[10]. In particular, [8] showed that both of strategies achieve the same diversity gain in Nakagami-fading environments. However, with the AF protocol, only maximal-ratio combining (MRC) scheme is always investigated and other combining schemes are seldom discussed.

This paper is concerned with wireless AF relay networks that deploy multiple parallel relays communicating with the destination over orthogonal channels in the second phase. We propose and analyze a protocol for relay selection and HDD at the destination based on double SNR thresholds. One SNR threshold is used to select retransmitting relays: a relay retransmits if its received SNR is larger than a threshold, otherwise it remains silent. The other threshold is used at the destination so that the destination makes a HDD for on the receiver signal if its SNR is higher than the threshold, and does nothing (or declares an erasure) otherwise. Finally, the binary decision is made with the simple majority voting rule of the hard decisions. We focus on the cooperative diversity analysis for the case that the destination does not know the exact or average SNRs of the source-relay links. Our analysis shows that the full diversity order can be also achieved for the dual-hop cooperative networks even with HHDs at the destination.

The paper is organized as follows. Section 2 describes the cooperative relaying model for AF protocol and the proposed relay selection and HDD protocol. Section 3 presents cooperative diversity order analysis. Simulation results are provided in Section 4. Finally, Section 5 concludes the paper.

2. System Model

Consider a wireless cooperative relay with $R+2$ nodes. The system has one source node, one destination node, and R relay nodes. Each node is equipped with only one antenna and works in a half-duplex mode. For simplicity, we first assume that there is no direct link from the source to destination. All channel links are assumed to be quasi-static and mutually independent, which means that the channels are constant within one transmission duration, but vary independently over different transmission durations. Furthermore, it is assumed that the destination knows the channel state information (CSI) of every relay-destination link and each relay knows the CSI of its source-relay link.

Information transmission over a wireless relay network is accomplished in two phases. In the first phase, signals are broadcasted by the source to the relays. In the second phase, each relay decides independently whether its detection is reliable by comparing its received SNR to a threshold value. If the detection is considered to be reliable, the relay retransmits by the AF protocol. Otherwise, it remains silent. It is also assumed that the destination knows whether a relay retransmits in the second phase, for example, by looking for a flag bit. For each received signal from the reliable relays, the destination only makes a binary decision detection when the relay-destination link is considered to be reliable, i.e., the received SNR of the link is higher than a second threshold value. Otherwise, the destination does nothing (erasure mode). The destination then makes a final binary decision by a simple majority voting on multiple HDDs.

Information transmission over a wireless relay network is accomplished in two phases. In the first phase, source broadcasts a modulated signal s to the relays. The received signal at the i th Relay $(i=1,\cdots,R)$ is expressed as

$$r_i=\sqrt{E_s}f_i s+v_i \tag{1}$$

Where s has unit power, thus E_s is the transmit power; f_i is the channel gain between the source and the i th relay, following Rayleigh distribution with the second moment $N_i^{(1)}$; and v_i is the complex additive white Gaussian noise (AWGN) with zero mean and unit variance.

In the second phase, with the AF protocol, the i th reliable relay amplifies the received symbol r_i with a gain G_i. If the destination also knows the CSI of the source-relay channel, the gain is usually set to $G_i=\sqrt{1/(|f_i|^2 E_s+1)}$ [8]. However, as we are interested in the case that such CSI is not available at the destination, the gain is modified to

$$G_i=\frac{f_i^*}{|f_i|^2\sqrt{E_s}}\cdot G \tag{2}$$

where $(\cdot)^*$ denotes conjugation, and G is a constant which is related to the SNR threshold and will be defined in next section. In essence (2) corrects the phase error of the received signal introduced by the source-relay link. The received signal at the destination from the i th relay can be written as

$$y_i=\sqrt{E_i}g_i(G_i r_i)+\omega_i \tag{3}$$

where E_i is the transmit power of the i th relay; g_i is the channel gain between the i th relay and the destination, following Rayleigh distribution with second moment $N_i^{(2)}$; and ω_i denotes the AWGN at the destination with zero mean and unit variance.

It is assumed that all the random variables $\{f_i\}_{i=1}^R$, $\{g_i\}_{i=1}^R$, $\{v_i\}_{i=1}^R$, and $\{\omega_i\}_{i=1}^R$ are independent of each other. Furthermore, for simplicity of analysis, we assume that $N_1^{(1)} = \cdots = N_R^{(1)} = N^{(1)}$, $N_1^{(2)} = \cdots = N_R^{(2)} = N^{(2)}$ and $E_1 = \cdots = E_R = E_s = E$.

3. Diversity Analysis for AF Protocol

3.1 Performance for the i th Relay Link

We first consider the performance of the i th relay link which is a cascade of the source-to-i th relay link and i th relay-to-destination link. Denote the instantaneous SNRs of these two individual links by $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$ and they are given by

$$\gamma_i^{(1)} = |f_i|^2 E,\ \gamma_i^{(2)} = |g_i|^2 E \tag{4}$$

Furthermore, the instantaneous SNR at the destination can be written as [8]

$$\gamma_i = \frac{\gamma_i^{(1)}\gamma_i^{(2)}}{\gamma_i^{(1)} + \gamma_i^{(2)} + 1} \tag{5}$$

Let $p_b(\gamma_i)$ represent the bit error rate (BER) of i th relay link with respect to the SNR γ_i. For a general modulation scheme, it can be approximated as [12]

$$p_b(\gamma_i) \approx \alpha Q(\sqrt{\beta\gamma_i}) \tag{6}$$

where $\alpha > 0$ and $\beta > 0$ depend on the type of modulation. For instance, with BPSK, $\alpha = 1$ and $\beta = 2$ give the exact BER.

Now let Θ_1 and Θ_2 denote the two SNR thresholds used at the relays and destination, respectively. Let $F_J(\cdot)$ and $f_J(\cdot)$, respectively denote the cumulative distribution function (cdf) and the probability density function (pdf) of the random SNR $\gamma_i^{(j)}, j = 1, 2$. Then the probability that the i th relay link is unreliable can be expressed as

$$p_u = 1 - [1 - F_1(\Theta_1)][1 - F_2(\Theta_2)] \tag{7}$$

With Rayleigh fading channels, $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$ are exponential random variables with mean values $N^{(1)}E$ and $N^{(2)}E$, respectively. Therefore

$$F_1(\Theta_1) = 1 - e^{-\Theta_1/(N^{(1)}E)} \tag{8}$$

$$F_2(\Theta_2) = 1 - e^{-\Theta_2/(N^{(2)}E)} \tag{9}$$

Let

$$\tilde{\gamma}_i = \frac{\Theta_1\gamma_i^{(2)}}{\Theta_1 + \gamma_i^{(2)} + 1} \tag{10}$$

To facilitate the detection operation at the destination, we now define G in (2) as

$$G = \sqrt{\frac{\Theta_1}{\Theta_1 + 1}} \tag{11}$$

When $\gamma_i^{(1)} \ge \Theta_1$, we can have that $|G_i| \le 1$. With the gain G_i in (2), if the maximum likelihood detection is performed, the instantaneous output SNR at the destination can be shown to be (see Appendix A)

$$\hat{\gamma}_i = \frac{\Theta_1\gamma_i^{(2)}}{\Theta_1 + \gamma_i^{(2)}(\Theta_1/\gamma_i^{(1)}) + 1} \tag{12}$$

When $\gamma_i^{(1)} \ge \Theta_1$, it is simple to see that

$$\hat{r}_i \ge \tilde{r}_i \tag{13}$$

Thus the average BER at the destination for the i th relay link under the reliable condition, i.e., $\gamma_i^{(1)} \ge \Theta_1$ and $\gamma_i^{(2)} \ge \Theta_2$, is written as

$$\begin{aligned} P_b &= \int_{\Theta_1}^{\infty}\int_{\Theta_2}^{\infty} p_b(\gamma_i^{(1)}, \gamma_i^{(2)}) f_1(\gamma_i^{(1)}) f_2(\gamma_i^{(2)}) \mathrm{d}\gamma_i^{(1)} \mathrm{d}\gamma_i^{(2)} \\ &= \int_{\Theta_1}^{\infty}\int_{\Theta_2}^{\infty} p_b(\hat{\gamma}_i) f_1(\gamma_i^{(1)}) f_2(\gamma_i^{(2)}) \mathrm{d}\gamma_i^{(1)} \mathrm{d}\gamma_i^{(2)} \\ &\le \int_{\Theta_1}^{\infty}\int_{\Theta_2}^{\infty} p_b(\tilde{\gamma}_i) f_1(\gamma_i^{(1)}) f_2(\gamma_i^{(2)}) \mathrm{d}\gamma_i^{(1)} \mathrm{d}\gamma_i^{(2)} \\ &\le \int_{\Theta_2}^{\infty} p_b(\hat{\gamma}_i) f_2(\gamma_i^{(2)}) \mathrm{d}\gamma_i^{(2)} (1 - F_1(\Theta_1)) \end{aligned} \tag{14}$$

where $p_b(\gamma_i^{(1)}, \gamma_i^{(2)})$ represents the BER of i th relay link with respect to the SNRs $\gamma_i^{(1)}$ and $\gamma_i^{(2)}$.

In order to present the asymptotical analysis for P_u and P_b, let us introduce the following two common notations. For two positive functions $a(x)$ and $b(x)$, $a(x) \sim b(x)$ means that $\lim_{\chi\to\infty} a(x)/b(x) = 1$, whereas $a(x) = O(b(x))$ means that $\limsup_{x\to\infty} a(x)/b(x) < \infty$. Furthermore, similar to [11], we shall define the two SNR thresholds as follows:

$$\Theta_1 = c_1 N^{(1)} \log E \tag{15}$$

$$\Theta_2 = c_2 N^{(2)} \log E \tag{16}$$

where c_1 and c_2 are two positive constants, whose values are discussed at the end of this subsection.

With the above definitions of the two SNR thresholds and as the SNR $E \to \infty$, one has

$$P_u = (1 - e^{-c_1 \log E/E} \cdot e^{-c_2 \log E/E})$$

$$= 1 - e^{-c\log E/E} \sim c\cdot\frac{\log E}{E} \qquad (17)$$

where $c = c_1 + c_2$.

Next, for the asymptotic analysis of P_b, define

$$\gamma_\Theta = \frac{\Theta_1\Theta_2}{\Theta_1 + \Theta_2 + 1} \qquad (18)$$

when $\gamma_i^{(1)} \geq \Theta_1$ and $\gamma_i^{(2)} \geq \Theta_2$, it is easy to show that

$$\gamma_i \geq \tilde{\gamma}_i \geq \gamma_\Theta \qquad (19)$$

With the help if (19) and the bound $Q(x) \leq \frac{1}{2}e^{-x^2/2}$, one has

$$P_b \leq \alpha Q(\sqrt{\beta r_\Theta})[1 - F_1(\Theta_1)][1 - F_2(\Theta_2)]$$
$$\leq \frac{\alpha}{2}e^{-\beta r_\Theta/2}\cdot e^{-c\log E/E} \leq \frac{\alpha}{2}e^{-\beta r_\Theta/2} \qquad (20)$$

As $P_u \sim c\cdot\frac{\log E}{E}$, in order to achieve the full diversity order, it requires that P_b at least decays as $O(1/E^2)$. This then implies that γ_Θ needs to satisfy

$$\beta r_\Theta/2 \geq 2\log E \qquad (21)$$

Since $\gamma_\Theta \geq \min\{\Theta_1/2, \Theta_2/2\}$, the constants $c_j\ (j = 1,2)$ need to satisfy

$$c_j \geq \frac{8}{\beta}\cdot\frac{1}{N^{(j)}},\ j = 1,2 \qquad (22)$$

Let $\Theta_{\min} = \min\{\Theta_1, \Theta_2\}$. Using (21) and (22), P_b in (20) can be bounded as

$$P_b \leq \frac{\alpha}{2}e^{-(\beta\Theta_{\min}/4)} \leq \frac{\alpha}{2}e^{-2\log E} = \frac{\alpha}{2}\cdot\frac{1}{E^2} \qquad (23)$$

Which confirms that $P_b \sim O(1/E^2)$.

3.2 Average Bit Error Probability and Diversity Order

Let $\overline{P}_B$ denote the overall average BER for the proposed cooperative relay network. Recall that the diversity order is defined as

$$d = -\lim_{E\to\infty}\frac{\log \overline{P}_B}{\log E} \qquad (24)$$

In the following, it is shown that an upper bound on the BER gives $d = R$ for the AF protocol. This implies that the relay network can achieve the full diversity.

Let $P_b(m,k)$ be the BER of the majority voting on the HDDs under the conditions that (i) among all R relays, there are m relays making binary decisions and $R - m$ relays making erasure decisions, and (ii) among m relays making binary decisions, there are k relays making correct decisions (i.e., $m - k$ relays making error decisions). Obviously, if $k > m - k$ the final binary decision is correct and thus $P_b(m,k) = 0$. On the other hand, if $k < m - k$ the final binary decision is wrong and thus $P_b(m,k) = 1$. If it happens that $k = m - k$, the destination makes the final binary decision by chance and hence $P_b(m,k) = 1/2$. Therefore, the conditional BER $P_b(m,k)$ can be written as

$$P_b(m,k) = \begin{cases} 0, & k > m-k \\ 1/2, & k = m-k \\ 1, & m-k > k \end{cases} \qquad (25)$$

It should be noted that when $m = k = 0$, no information data is sent over the wireless relay network. In such a case, the conditional BER can be set to $\frac{1}{2}$ for further unified analysis.

The overall BER can be written as

$$\overline{P}_B = \sum_{m=0}^{R}\sum_{k=0}^{m}\binom{R}{m}\binom{m}{k}P_u^{R-m}P_b^{m-k}(1 - P_u - P_b)^k P_b(m,k) \qquad (26)$$

Since $(1 - P_u - P_b)^k \leq 1$, $\overline{P}_B$ can be upper bounded as follows:

$$\overline{P}_B \leq \sum_{m=0}^{R}\sum_{k=0}^{[m/2]}\binom{R}{m}\binom{m}{k}P_u^{R-m}P_b^{m-k} \qquad (27)$$

Here $\lfloor m/2 \rfloor = m/2$ if m is even, and $\lfloor m/2 \rfloor = (m-1)/2$ if m is odd. It follows from (17) and (23) that

$$\overline{P}_B \leq \sum_{m=0}^{R}\sum_{k=0}^{[m/2]}\binom{R}{m}\binom{m}{k}\left(\frac{c\log E}{E}\right)^{R-m}\left(\frac{\alpha}{2E^2}\right)^{m-k} \qquad (28)$$

Since $R \leq R - m + 2(m-k) = R + m - 2k$ for $k \leq [m/2]$, one has

$$\overline{P}_B \leq \left(\frac{\log E}{E}\right)^R \sum_{m=0}^{R}\sum_{k=0}^{[m/2]}\binom{R}{m}\binom{m}{k}c^{R-m}\left(\frac{\alpha}{2}\right)^{m-k}$$
$$\leq q\left(\frac{\log E}{E}\right)^R \qquad (29)$$

Where q is a positive constant equal to

$$q = \sum_{m=0}^{R}\sum_{k=0}^{[m/2]}\binom{R}{m}\binom{m}{k}c^{R-m}(\alpha/2)^{m-k}$$
$$\leq (c + 1 + \alpha/2)^R \qquad (30)$$

Finally, the diversity order is calculated as

$$d = \lim_{E\to\infty}\frac{\log \overline{P}_B}{\log E} = R \qquad (31)$$

Since there is no the direct link from the source to the destination, it is possible that an outage event occurs for the network when no information is actually sent to the destination. Based on (17), the outage probability is equal to

$$\overline{P}_{out} = P_u^R \sim (c \cdot \frac{\log E}{E})^R \quad (32)$$

Obviously, when $E \to \infty$, $\overline{P}_{out} \to 0$. Therefore, at high SNR region the outage event has a negligible influence on the BER performance.

4. Numerical Results and Comparison

This section provides simulation results to illustrate the performance of the proposed method with SNR thresholds and hard-decision detections. For simplicity, only BPSK modulation (which means that $\alpha = 1$ and $\beta = 2$) is considered. In all the simulation curves, SNR denotes the total power $(R+1)E$ since the variance of AWGN is set to one. Moreover, we set $N^{(1)} = 8$ and $N^{(2)} = 4$.

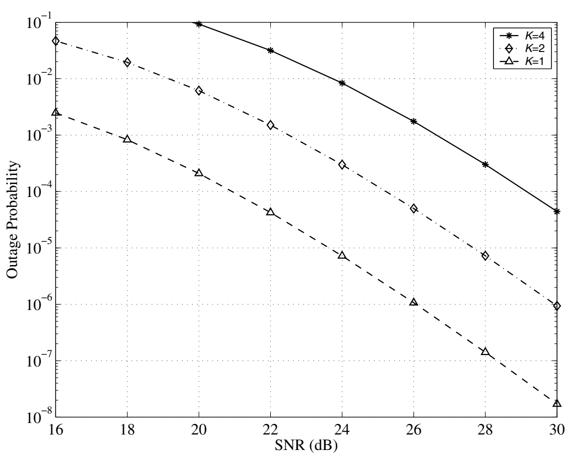

Fig.1. Outage performance comparison for different SNR threshold values.

We first consider the influence of SNR thresholds on the network outage performance. Fig. 1 plots the outage probability based on (32) for three different pairs of thresholds (c_1, c_2) . In particular, we let $R = 6$ relays and set

$$c_j = K \cdot \frac{8}{\beta} \cdot \frac{1}{N^{(j)}}, j = 1,2 \quad (33)$$

for $K = 1,2,4$. Note that the resulting threshold values meet the inequality in (22). It can be seen that the network outage performance becomes significantly worse as K increases. Based on this fact, in all the subsequent simulations the smallest threshold values that meet the stated conditions (such as the one in (22)) are always selected.

Fig. 2 compares the BER performance for three different numbers of relays, namely $R = 4,6,8$. It can be observed that the system BER performance with SNR thresholds and HDDs improves as SNR or R increases.

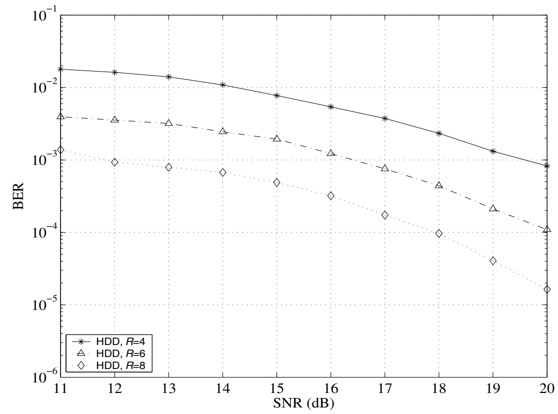

Fig.2. BER performance comparison for different numbers of Relays.

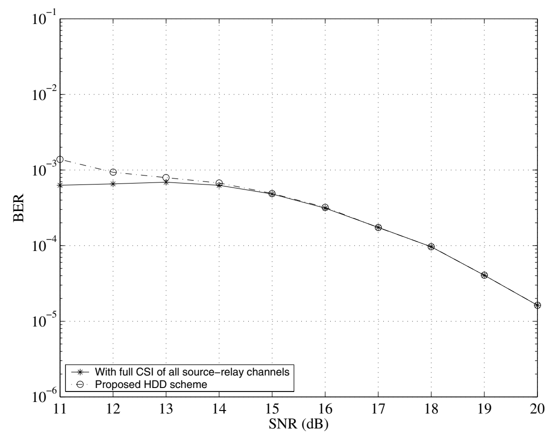

Fig.3. Performance comparison between the proposed HDD scheme and the scheme that has the destination know CSI of all the source-relay links.

Finally, we compare the BER performance between two different detection schemes at the destination when $R = 8$. One scheme is when the destination perfectly knows the CSI from the source to all the relays and uses it for the maximum likelihood detection. The other is our proposed HDD scheme that does not require the destination to know the CSI from the source to relays, but it knows the SNR thresholds used at the relays. Fig 3 clearly shows that, as the SNR increases, the performance of the proposed HDD scheme closely approaches the performance of the scheme that makes use of the CSI of all source-relay links.

5. Conclusion

In this paper, we have studied the diversity performance of wireless cooperative AF relay networks with SNR thresholds and HHDs. We first derived end-to-end BER and outage probability, and then presented diversity analysis. The proposed HHD scheme is very simple to implement and yet achieves the full diversity order if appropriate threshold values are set. In particular, the proposed HDD scheme does not require the destination to know the exact or average SNRs of the source-relay links.

Appendix

A. *Proof of the Instantaneous SNR Expression in (12) with AF Protocol*

First, substituting (1) into (3) yields

$$y_i = \sqrt{E}_i g_i G_i \sqrt{E}_s f_i s + \sqrt{E}_i g_i G_i v_i + \omega_i \qquad (34)$$

Using (11) in (2) and then substituting (2) into (34), (34) becomes

$$y_i = \sqrt{E} g_i \sqrt{\frac{\Theta_1}{\Theta_1 + 1}} \cdot s + \sqrt{E} g_i \frac{f_i^*}{|f_i|^2 \sqrt{E_s}} \sqrt{\frac{\Theta_1}{\Theta_1 + 1}} \cdot v_i + \omega_i \qquad (35)$$

where we have used $E_s = E_i = E$ as assumed. Based on the above expression, the instantaneous SNR at the destination can be written as

$$\hat{\gamma}_i = \frac{E|g_i|^2 \cdot \frac{\Theta_1}{\Theta_1 + 1}}{E|g_i|^2 \cdot \frac{1}{|f_i|^2 E_s} \cdot \frac{\Theta_1}{\Theta_1 + 1} + 1} = \frac{\Theta_1 \gamma_i^{(2)}}{\Theta_1 + \gamma_2^{(2)} (\Theta_1 / \gamma_1^{(1)}) + 1} \qquad (36)$$

since $\gamma_i^{(2)} = E|f_i|^2$ and $\gamma_i^{(2)} = E|g_i|^2$. This concludes the proof.

Acknowledgments

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